

Max. Marks: 80

Time: 3 hrs.

N.B. : 1. Q1 is compulsory

2. Attempt any three questions from Q2 to Q6.
3. Figures to the right indicate full marks.

Q1. (a) A r.v. X assumes the values $-3, -2, -1, 0, 1, 2, 3$ such that 5

$$P(X = 1) = P(X > 1) = P(X \leq 0) \text{ and } 2P(X = -3) = P(X = -2) = 5P(X = -1),$$

$$P(X = 1) = 3P(X = 2) = 2P(X = 3). \text{ Find the pmf and the distribution of } X.$$

(b) The following calculations have been made for closing prices of 12 stocks (x) on the Mumbai stock exchange on a certain day, along with the volume of sales in thousands of shares (y). 5

$$\sum x = 580, \sum y = 370, \sum x^2 = 41658, \sum xy = 11494, \sum y^2 = 17205$$

From these calculations, find the linear regression equation of volume of sales depending on stock price.

(c) Evaluate the integral $\int_C \frac{z-1}{z^2+3z+2} dz, \quad C: |z| = \frac{3}{2}.$ 5

(d) Convert the given set of vectors into an orthonormal basis using Gram Schmidt process of orthogonalization. $(2, -1, 1), (1, -1, 3), (1, 1, 2).$ 5

Q2. (a) Is the following a subspace of a given vector space with usual addition and scalar multiplication? Justify your answer. 6

$$(i) W = \{(x, y, z) \mid x, y, z \in \mathbb{R}, y = x + z\}$$

$$(ii) W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R}, \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0 \right\}$$

(b) A random variable X has the probability density function 6

$$f(x) = k x (9 - x^2), \quad 0 \leq x \leq 3. \text{ Find } k \text{ and mean of } X.$$

(c) Fit the least square line to the following data i) line of y on x ii) line of x on y . 8
Also find the correlation coefficient using the regression coefficients.

$$\begin{array}{ccccccc} x: & 65 & 63 & 67 & 64 & 68 & 62 & 70 \\ y: & 68 & 66 & 68 & 65 & 69 & 66 & 68 \end{array}$$

Q3. (a) At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a (i) woman (ii) man? 6

(b) Find the extremals of $\int_{x_1}^{x_2} \frac{(y')^2}{x^2} dx.$ 6

- (c) Find a singular value decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. 8
- Q4. (a) Find the usual inner product of the two vectors $(-4, 6, -1, 1)$, $(2, 1, -2, 9)$. 6
 Find the norm of each vectors. Are these vectors orthogonal to each other?
 Verify the triangle inequality and Cauchy Schwarz inequality.
- (b) Evaluate the following integrals using Cauchy Residue theorem, 6

$$\int_C \frac{1}{z^5} e^{z^2} dz, \quad |z|=1.$$
- (c) The marks of 1000 students in a semester examination of an Engineering 8
 college are distributed normally with mean 70% and standard deviation 5%.
 Estimate the number of students whose marks will be i) between 60% and
 75% ii) more than 75% iii) less than 68%.
- Q5. (a) Find all possible Laurent's series expansion of $\frac{2z+1}{z^2+5z+6}$ about the origin. 6
- (b) Find the extremals of $\int_0^1 xy + y^2 - 2y^2 y' dx$. 6
- (c) Reduce the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2 + 6x_1x_3 - 4x_2x_3$ to a diagonal 8
 form using a congruent transformation. Obtain the congruent transformation
 applied for the reduction. Find the rank, index signature and class value of the
 quadratic form.
- Q6. (a) An examination of 11 applicant for an accountant post was taken by a finance 6
 company. The marks obtained by the candidate in reasoning (x) and aptitude
 (y) test are given below. Calculate the rank correlation coefficient between the
 performance in the reasoning and aptitude test.
 x : 20 50 25 70 90 50 76 45 30 19 26
 y : 30 60 40 50 45 30 68 30 47 39 38
- (b) Evaluate using Cauchy integral formula, 6

$$\int_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz, \quad C: |z - 2 - 2i| = 3.$$
- (c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of 8

$$\int_0^1 2xy - y^2 - (y')^2 dx, \quad y(0) = 0, y(1) = 0.$$