

SE C-scheme choice based (R-19 'c' scheme) - Summer 2025

02.6.2025

CMPN

TIME: 03 HOURS

MAX. MARKS : 80

Note:

1. Question No. 1 is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

Q.1 Answer the following (Any four)

Marks

- a. Find the Laplace transform of $t \sin^3 t$.
- b. Calculate the Spearman's rank correlation coefficient R.

05

05

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

- c. Find the constants a, b, c, d, e if $f(z) = (ax^3 + bx^2y + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic.

05

- d. Find inverse Laplace transform of $\tan^{-1}\left(\frac{s+a}{b}\right)$.

05

Q.2 a. Evaluate by using Laplace transform of $\int_0^\infty \left(\frac{\sin 3t + \sin 2t}{te^t}\right) dt$.

06

- b. If the mean of the following distribution is 16 find m, n and variance

06

X	:	8	12	16	20	24
P (X=x)	:	$\frac{1}{8}$	m	n	$\frac{1}{4}$	$\frac{1}{12}$

- c. Obtain the Fourier expansion of $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in $(0, 2\pi)$

08

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ Q.3 a. Find the analytic function $f(z) = u + i v$ in terms of z if $u + v = e^x (\cos y + \sin y) + \frac{x-y}{x^2+y^2}$.

06

- b. Find the coefficient of regression and hence the equations of the lines of regression for the following data

06

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

- c. Using convolution theorem Find the inverse Laplace transform of

08

$$\frac{1}{(s^2 + 4s + 13)^2}$$

Q.4 a. Obtain Fourier series of $f(x) = |\sin x|$ in $((-\pi, \pi))$. **06**

b. If X denotes the outcome when a fair die is tossed, find the moment generating function of x and hence find the mean and variance of X . **06**

c. Evaluate by using Laplace transforms of $\int_0^\infty e^{-t} (t \int_0^t e^{-4u} \cos u \, du) dt$. **08**

Q.5 a. Find the orthogonal trajectories of family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$. **06**

b. Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. **06**

c. Fit a second-degree parabolic curve to the following data and estimate the Production in 1982. **08**

Year (X)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y)(in tons)	12	14	26	42	40	50	52	53

Q.6 a. Obtain half range Sine series for $f(x) = x - x^2$ in $0 \leq x \leq 1$. **06**

Hence show that $\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$

b. Show that the function $v = e^{2x}(y \cos 2y + x \sin 2y)$ is harmonic. And find its corresponding analytic function $f(z) = u + iv$. **06**

c. Find the value of k if the function $f(x) = kx^2(1 - x^3)$, $0 \leq x \leq 1$ **08**

$f(x) = 0$ otherwise.

Is a probability density function. Also find $p(0 \leq x \leq \frac{1}{2})$ find mean and variance.