University of Mumbai

Examinations Summer 2022

Time: 2hour 30 minutes Max. Marks: 80

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Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	Evaluate $\int_0^{1+i} (x^2 + iy)\,dz$ along the path $y = x$
Option A:	$\frac{-1+5i}{6}$
Option B:	6
Option C:	-1
Option D:	0
2.	The function $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ has
Option A:	simple pole at z = -2 & pole of order 2 at z = 1
Option B:	simple pole at z = 2
Option C:	simple pole at z = 0 & pole of order 1 at z = 2
Option D:	simple pole at z = 0
3.	If the line of regression of Y on X is (Y- 33) = (0.72)(X- 33) then the approximate value of Y for X = 38 is
Option A:	20
Option B:	37
Option C:	1
Option D:	10
4.	If for the non-repeated ranks $\sum D^2 = 8$ & n = 5 then the rank correlation coefficient (R) is
Option A:	0
Option B:	0.6
Option C:	7
Option D:	1
5.	Find k if pdf of rv X is

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

Option A:	1/5
Option B:	2
Option C:	1/49
Option D:	7
6.	Find E(X) if $f(x) = x \qquad 0 \le x \le 1$
Option A:	2
Option B:	1
Option C:	1/3
Option D:	0

7.	If u=(-2, 3, 4) and v=(3, -2, 3) then their dot product is
Option A:	3
Option B:	1
Option C:	2
Option D:	0
8.	If u = (2,1,0) then $\|u\|$
Option A:	$\sqrt{50}$
Option B:	$\sqrt{3}$
Option C:	$\sqrt{5}$
Option D:	1
9.	The matrix form of the Quadratic form $x^2 - 2y^2 + 3z^2 - 4xy + xz - 2yz$ is
Option A:	$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -2 & 1/2 \\ -2 & -2 & -1 \\ 1/2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Option B:	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & -2 & -1 \\ 1/2 & 1 & 3 \end{bmatrix}$
Option C:	$\begin{bmatrix} 1 & -2 & 1/2 \\ -2 & -2 & -1 \\ 1/2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix}$
Option D:	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix}$
10.	Find the extremals of $\int_{x_1}^{x_2} \frac{y'^2}{x^2} dx$
Option A:	$y = c_1 x^5 + c_2$
Option B:	$y = c_1 x + c_2$
Option C:	$y = c_1 x^3 + c_2$
Option D:	$y = c_1 x^6 + c_2$

Q2	**Solve any Four out of Six** **5 marks each**
A	Using residue theorem evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} dz$ where C is the circle $\|z\| = 2$
B	Fit a straight line to the following data X : 5 6 7 8 9 Y : 2 4 5 6 8
C	For a normal variate with mean 2.5 and standard deviation 3.5. Find the probability that i) $2 \le x \le 4.5$ ii) $-1.5 \le x \le 5.5$
D	Find a unit vector in R^3 orthogonal to both u = (1, 0, 1) and v = (0, 1, 1).

E	Reduce the quadratic form $6x^2+3y^2+14z^2+4xy+4yz+18xz$ to diagonal form using congruent transformations.
F	Find the extremals of $\int_0^1 (xy+y^2-2y^2y')\,dx$

Q3	**Solve any Four out of Six** **5 marks each**
A	Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle $\|z\|=2$
B	Given 6y = 5x + 90 ; 15x = 8y + 130. Find i) $\bar{x}$ and $\bar{y}$ ii) r
C	Three factories A, B, C produce 30%, 50% and 20 % of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A.
D	Construct an orthonormal basis of R^2 Using Gram-Schmidt process to S = { (3, 1), (2, 3) }
E	Reduce the quadratic form $x^2+2y^2+2z^2-2xy-2yz+xz$ to canonical form. Also find its rank & signature.
F	Find the extremals of $\int_{x_0}^{x_1} (2xy-y''^2)\,dx$

Q4	**Solve any Four out of Six** **5 marks each**
A	Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in the regions i) $\|z\|<1$ ii) $1<\|z-1\|<2$
B	Calculate R and r from the following data X : 12 17 22 27 32 Y : 113 119 117 115 121
C	Fit a poission distribution to the following data No. of deaths : 0 1 2 3 4 Frequencies : 123 59 14 3 1
D	Let V be a set of positive real numbers with addition and scalar multiplication defined as x + y = xy and kx = x^k Show that V is a vector space under this addition an scalar multiplication.
E	Find the singular value decomposition of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$
F	Using Rayleigh-Ritz method , solve the boundary value problem $\int_0^1 (2xy+y^2-y'^2)\,dx$; $0 \le x \le 1$ given y(0) = y (1) = 0