

(Time: 3 hours)

Max Marks: 80

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

- a) Solve $(2x^2 + 3y^2 - 7)x dx + (3x^2 + 2y^2 - 8)y dy = 0$ 5
- b) Solve $\frac{d^2y}{dx^2} - y = e^{2x} + \sin 2x$ 5
- c) Using Euler's method find the approximate value of y at $x = 0.5$ taking $h = 0.1$, $\frac{dy}{dx} = 2 + \sqrt{xy}$, $y(0) = 1$ 5
- d) Change the order of integration $I = \int_0^2 \int_{\sqrt{2x}}^2 f(x, y) dy dx$ 5

Q2.

- a) Solve $(D^3 - 2D + 4)y = 3x^2 - 5x$ 6
- b) Solve $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$ 6
- c) Evaluate $\int_0^6 e^x dx$ by using 8
- (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule

Q3.

- a) Using the Rule of DUIS prove that $\int_0^\infty \frac{\log(1+ax^2)}{x^2} dx = \pi\sqrt{a}$ 6
- where $a \geq 0$ hence deduce $\int_0^\infty \frac{\log(1+x^2)}{x^2} dx = \pi$
- b) Evaluate $\int \int xy dx dy$ over the region bounded by X-axis, line $x = 2a$ and the parabola $x^2 = 4y$ 6
- c) Evaluate $\int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} z dz dy dx$ 8

Q4.

- a) Find area of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ 6
- b) Using Runge-Kutta method of fourth order find the approximate value of y at $x = 0.1$ taking $h = 0.1$, $\frac{dy}{dx} = x + \sqrt{y}$ & $y(0)=1$ 6
- c) Evaluate $\int_0^3 \sqrt{3x - x^2} dx \cdot \int_0^\infty \frac{1}{(1+x^2)^{3/2}} dx$ 8

Q5.

- a) Evaluate $\int \int \int (x^2 + y^2 + z^2) dx dy dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$ 6
- b) Solve $(x^2 y^2 + 2)y dx + (2 - 2x^2 y^2)x dy = 0$ 6
- c) Solve by method of Variation of parameter
 $(D^2 - 2D + 1)y = e^x \sin x$ 8

Q6.

- a) Using Euler's modified method find the approximate value of y at $x = 1.2$ taking $h = 0.2$, $\frac{dy}{dx} = \log_e(x + y)$, $y(1) = 2$ correct upto 4 decimal places 6
- b) Find the length of cardioide $r = a(1 - \cos \theta)$ which lies outside the circle $r = a \cos \theta$. 6
- c) Change to polar coordinates and evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy dx$ 8
