University of Mumbai

Program: _First Year (All Branches) Engineering- SEM-I

Curriculum Scheme: Rev 2019

Engineering Mathematics-I

Question Bank

===

Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	The real part of $\cos^{-1}\left(\frac{2i}{3}\right)$ is
Option A:	π
Option B:	2 π
Option C:	- π
Option D:	π/2
2.	The Cartesian form of $4(\cos\pi/2 + i\ \sin\pi/2)$ is equal to
Option A:	2i
Option B:	-2i
Option C:	4i
Option D:	4
3.	Find the value of $\log(\sqrt{3}\ -\ i)$
Option A:	$log4 + i\frac{\pi}{6}$
Option B:	$log2 + i\frac{\pi}{6}$
Option C:	$log4 - i\frac{\pi}{6}$
Option D:	$log2 - i\frac{\pi}{6}$
4.	$If\ coshx = sec\theta\ \ then\ x\ is$
Option A:	$\log(\sec\theta - \sin\theta)$
Option B:	$\log(\sec\theta + \tan\theta)$
Option C:	$\log(\tan\theta - \sec\theta)$
Option D:	$\log(\cos\theta + \sin\theta)$
5.	$If\ z = x^2 + y^2, x = cost, y = sint\ , then\ the\ value\ of\ \frac{dz}{dt}\ at\ t = \pi$
Option A:	1
Option B:	-1
Option C:	0
Option D:	π
6.	$If\ u = \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}\ \ then\ \ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\ is$
Option A:	$\frac{u}{2}$
Option B:	$\frac{-u}{2}$
Option C:	$2u$
Option D:	$\sqrt{2}\,u$
7.	Stationary point is a point where f(x,y) has

Option A:	$\frac{\partial f}{\partial x} = 0$
Option B:	$\frac{\partial f}{\partial y} = 0$
Option C:	$\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$
Option D:	$\frac{\partial f}{\partial x} < 0$ $\frac{\partial f}{\partial x} > 0$
8.	$The\ n^{th}\ derivative\ of\ sin2x\ cos3x\ \ is$
Option A:	$\frac{1}{2}\left(5^n sin\left(5x+\frac{n\pi}{2}\right) - sin\left(x+\frac{n\pi}{2}\right)\right)$
Option B:	$\frac{1}{2}\left(5^n sin\left(5x+\frac{n\pi}{2}\right) + sin\left(x+\frac{n\pi}{2}\right)\right)$
Option C:	$\frac{1}{2}\left(5^n sin\left(5x+\frac{n\pi}{2}\right) - cos\left(x+\frac{n\pi}{2}\right)\right)$
Option D:	$\frac{1}{2}\left(5^n sin\left(5x+\frac{n\pi}{2}\right) + cos\left(x+\frac{n\pi}{2}\right)\right)$
9.	For the unitary matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$, find A-1
Option A:	$\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
Option B:	$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Option C:	$\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
Option D:	$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$
10.	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then rank of A is
Option A:	2
Option B:	3
Option C:	1
Option D:	0

Descriptive Questions

1.	Prove that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$
2.	Prove that $log\left(1+e^{i\theta}\right) = \log\left(2\,cos\frac{\theta}{2}\right) + i\frac{\theta}{2}$
3.	Find the Rank of the following matrix by reducing to Normal Form $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$
4.	Find a, b, c if A is orthogonal matrix where $A = \frac{1}{3}\begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$. Hence find inverse of A.
5.	Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum using Lagrange's method.
6.	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$
7.	Solve: $x^4 - x^3 + x^2 - x + 1 = 0$
8.	If $\cosh x = \sec\theta$ Prove that $\theta = \frac{\pi}{2} - 2\tan^{-1}(e^{-x})$
9.	Express the matrix $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ as a sum of Hermitian and skew Hermitian matrix.
10.	If $\cos^{-1}\frac{y}{b} = log(\frac{x}{n})^n$, then prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$
11.	Find the n^{th} derivative of $\frac{x}{1+3x+2x^2}$
12.	If $u = sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u\ \cos 2u}{4cos^3 u}$,
13.	Test for consistency the following system & solve them if consistent $x_1 - 2x_2 + x_3 - x_4 = 2$ $x_1 + 2x_2 + 2x_4 = 1$ $4x_2 - x_3 + 3x_4 = -1$
14.	Prove that $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1}\cos\frac{n\pi}{3}$
15.	Prove that $\sinh^{-1}(\tan\theta) = log\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$
16.	If $\tanh x = 0.5$, prove that $\sinh 2x = \frac{4}{3}$.
17.	If $z = x^4y^2\sin^{-1}\frac{x}{y} + log\frac{x}{y}$, Find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$
18.	Examine the function $u = x^3y^2(12 - 3x - 4y)$ For extreme values.
19.	Solve the equation $x^7 + x^4 + x^3 + 1 = 0$
20.	Find the values of $\sin h\ logi$ and $\cos h\ logi$
21.	If $u = f(x^n - y^n, y^n - z^n, z^n - x^n)$ $P.T$ $\frac{1}{x^{n-1}}\frac{\partial u}{\partial x} + \frac{1}{y^{n-1}}\frac{\partial u}{\partial y} + \frac{1}{z^{n-1}}\frac{\partial u}{\partial z} = 0$
22.	Find the extreme value of the function $x^2y - 3x^2 - 2y^2 - 4y + 3$

23.	Reduce to normal form and find its rank given A = $\begin{pmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{pmatrix}$
24.	Separate into real and imaginary parts $tan^{-1}(\alpha + i\beta)$
25.	If $x + iy = \cot\left(\frac{\pi}{6} + i\alpha\right)$ $P.T$ $x^2 + y^2 - 2\frac{x}{\sqrt{3}} = 1$
26.	Find all the values of $(1 - i\sqrt{3})^{1/4}$
27.	If $u = tan^{-1}(\frac{x^2+y^2}{x-y})$ P.T $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2sin^3u\, cosu$
28.	For what value of λ the equations $x + 2y + z = 3, x + y + z = \lambda$, $3x + y + 3z = \lambda^2$ have a solution and solve them completely in each case.
29.	If $y^{1/m} + y^{-1/m} = x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)\, y_n = 0$
30.	If $z = x^2y + y^2, x = \log t, y = e^t$ find $\frac{dz}{dt}$ at $t = 1$
31.	Find the real part of the principal value of $(1 + i)^{\log i}$
32.	Expand $cos^7\theta$ in a series of cosines of multiples of θ
33.	If z = $tan^{-1}(\frac{y}{x})$, find $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$
34.	If $y = x^n \log x$, find y_{n+1}
35.	If A = $\begin{pmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{pmatrix}$ find non singular matrices Pand Q such that PAQ is in normal form and find its rank
36.	Express the following skew Hermitian matrix A as P + i Q where P is real and skew symmetric and Q is real and symmetric given A = $\begin{pmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{pmatrix}$
37.	If $\sin(\theta + i\emptyset) = \tan\alpha + i\sec\alpha$ then prove that $\cos 2\theta \cosh 2\emptyset = 3$
38.	Prove that $\frac{\sin 6\theta}{\sin 2\theta} = 16cos^2\theta - 16cos^2\theta + 3$
39.	If $z = f(x, y), x = u\cosh v , y = u\sinh v$ then prove that $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2}\left(\frac{\partial z}{\partial v}\right)^2$
40.	Determine the values of k for which the following equations are consistent. Also solve for any one values of k. x+2y+z=3 , x+y+z =k , 3x+y+3z=k^2