

S.E. INFT SEM III C-Scheme June 2026

12

TIME: 03 HOURS

MAX. MARKS : 80

04/06/26.

Note:

1. Question No. 1 is compulsory.
2. Attempt any three questions out of remaining five questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

- Q.1 Answer the following Marks
- a. Find the Laplace transform of $\frac{\cos 2t \sin t}{e^{-t}}$ 05
- b. Calculate Karl Pearson's coefficient of correlation r between x and y . 05
- $X : 28, 45, 40, 38, 35, 33, 40, 32, 36, 33$
- $Y : 23, 34, 33, 34, 30, 26, 28, 31, 36, 35$
- c. Find the constants a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic function 05
- d. Find inverse Laplace transform of $\log\left(1 + \frac{a^2}{s^2}\right)$ 05
- Q.2 a. Prove that $\int_0^\infty e^{-t} \left(\frac{\sin 2t + \sin 3t}{t}\right) dt = \frac{3\pi}{4}$. 06
- b. The probability density function of a random variable X is 06
- | | | | | | | | |
|----------|-----|------|------|------|------|-------|-------|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(X=x)$ | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |
- Find (i) k (ii) $p(X < 4)$ (iii) $p(3 < X \leq 6)$.
- c. Find the Fourier Series to represent $f(x) = x^2$ in $(0, 2\pi)$. 08
- Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
- Q.3 a. Find an analytic function $f(z) = u + iv$ where 06
- $u = \frac{x}{2} \log(x^2 + y^2) - y \tan^{-1}\left(\frac{y}{x}\right) + \sin x \cos y$.
- b. Given $6y = 5x + 90$, $15x = 8y + 130$, $\sigma^2 x = 16$ find 06
- (i) mean of x and y (ii) r (iii) σy .
- c. Using convolution theorem Find the inverse Laplace transform of 08
- $\frac{s+3}{(s^2 + 6s+13)^2}$

- Q.4 a Obtain Fourier series of $f(x) = 1 - x^2$ in $((-1,1))$. 06
 b A random variable X has the probability distribution. b

X	0	1	2	3
P(X =x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Find the moment generating function of X and then find mean and variance.

- c. Evaluate by using Laplace transforms of 8
 $\int_0^\infty e^{-t} \left(\int_0^t u^2 \sin hu \cos hu du \right) dt$.
- Q.5 a Find the orthogonal trajectories of family of curves 06
 $3x^2y + 2x^2 - y^3 - 2y^2 = c$.
- b Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. 06
- c. Fit a second-degree parabolic curve to the following data and estimate the 08
 production in 1982

Year x	1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	12	14	26	42	40	50	52	53

- Q.6 a Obtain half range Cosine series for $f(x) = x(\pi - x)$ in $0 < x < \pi$. 06

Hence show that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$

- b Show that the function $v = e^x (x \sin y + y \cos y)$ satisfies Laplace equation 06
 and its corresponding analytic function and its harmonic conjugate.

- c Find the value of k if the function $f(x) = kx^2(1 - x^3)$, $0 \leq x \leq 1$ 08
 $f(x) = 0$, otherwise .

is a probability density function. Also find $p(0 \leq x \leq \frac{1}{2})$, find mean and variance.
