

FE Sem - I (C-Scheme) June 2026

(Time: 3 hours)

Max.Marks:80

- N.B (1) Question No.1 is compulsory
 (2) Answer any three questions from Q.2 to Q.6
 (3) Figures to the right indicate full marks

- 1 a) Express $A = \begin{bmatrix} 1+i & i \\ -i & 2-i \end{bmatrix}$ as sum of Hermitian and skew Hermitian matrices. 5
- b) If $z = \tan^{-1} \left(\frac{y}{x} \right)$, find $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ 5
- c) Find the value of $\left(\frac{-1+i\sqrt{3}}{2} \right)^{15} + \left(\frac{-1-i\sqrt{3}}{2} \right)^{15}$ 5
- d) Find the value of $\log(3+4i) + \log(3-4i)$ 5
- 2 a) Find all the values of $(1+i)^{\frac{1}{3}}$, and show their continued product is $1+i$. 6
- b) Find real and imaginary parts of $\cos^{-1} \left(\frac{3i}{4} \right)$ 6
- c) If $u = f(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ 8
- 3 a) Check if $A = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$ is orthogonal, if yes find A^{-1} . 6
- b) If $y = \frac{x}{(x+1)^4}$, find y_n 6
- c) Using Newton-Raphson method, find approximate value of $\sqrt[3]{150}$. 8
- 4 a) Find the extreme values of $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ 6
- b) Using De-Moivre's theorem, prove that $\frac{\sin 5\theta}{\sin \theta} = 16\cos^4 \theta - 12\cos^2 \theta + 1$ 6
- c) If $y = a \cos(\log x) + b \sin(\log x)$, then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$ 8
- 5 a) Test the consistency of the equations and solve if consistent $2x + y + z = 4, x - y + 3z = 3, 4x - y - z = 2$ 6
- b) Solve the following equations by Gauss-Seidel Method $28x + 4y - z = 32, 2x + 17y + 4z = 35, x + 3y + 10z = 24$ 6
- c) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ 8
- 6 a) Reduce the matrix to normal form, hence find its rank. $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$ 6
- b) Prove that $\log(\sec x) = \frac{x^2}{2} + \frac{1}{3} \frac{x^4}{4} + \frac{2}{15} \frac{x^6}{6} + \dots$ 6
- c) If $\sin(\theta + i\phi) = e^{i\alpha}$, then prove that $\cos^4 \theta = \sin^2 \alpha = \sinh^4 \phi$ 8
