

(TIME: 03 HOURS)

(MAX. MARKS : 80)

Note:

1. Question No. 1 is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

- Q.1 Answer the following Marks
- a. If matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ find Eigen values of $A^2 - 2A + I$. 05
- b. Evaluate $\int_0^{1+i} z^2 dz$ along (i) the parabola $x = y^2$ (ii) the line $y = x$. 05
- c. Determine all basic solutions and optimal basic feasible solution to the following problem. 05
- Max. $z = x_1 - 2x_2 + 4x_3$
- Subject to $x_1 + 2x_2 + 3x_3 = 7, 3x_1 + 4x_2 + 6x_3 = 15,$
- d. Find the z-transform of $f(k) = a^k, k \geq 0$. 05
- Q.2 a. Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$ 06
- b. The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviations from the means are 26.94 and 18.73 respectively. can the samples be considered to have drawn from the same population ? 06
- c. Solve the L.P.P by simplex method. 08

$$\text{Maximize } z = 100x_1 + 50x_2 + 50x_3$$

$$\text{Subject to } 4x_1 + 3x_2 + 2x_3 \leq 10$$

$$3x_1 + 8x_2 + x_3 \leq 8$$

$$4x_1 + 2x_2 + x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0.$$

- Q.3 a. Find the relative maximum or minimum of the function 06
 $Z = x_1^2 + x_2^2 + x_3^2 - 6x_1 - 10x_2 - 14x_3 + 103.$
- b. If $f(k) = \frac{1}{3^k}$ and $g(k) = \frac{1}{5^k}$, then find the Z-transform of 06
 $\{f(k) * g(k)\}.$
- c. Obtain Taylor's and Laurent's expansion of $f(z) = \frac{z-1}{(z+1)(z-3)}$ indicating 08
 the regions of convergence.

- Q.4 a. Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} 06
 where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}.$$
- b. A die was thrown 132 times and the following frequencies were observed. 06
 No. obtained : 1, 2, 3, 4, 5, 6. Total
 Frequency : 15, 20, 25, 15, 29, 28. 132
 Test the hypothesis that the die is unbiased.
- c. Using the Kuhn-Tucker conditions to solve the N.L.P.P 08
 Maximize $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$
 Subject to $2x_1 + 5x_2 \leq 98;$
 $x_1, x_2 \geq 0$

- Q.5 a. Evaluate $\int_C \frac{2z-1}{z(2z+1)(z+2)} dz$ using Cauchy's residue theorem, 06
 where C is the circle $|z| = 1$.
- b. Using the method of Lagrange's multiplier solve the N.L.P.P 06
 Optimize $z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23.$
 Subject to $x_1 + x_2 + x_3 = 10. \quad x_1, x_2, x_3 \geq 0.$

- c In an intelligence test administered to 1000 students, the average was 42 and standard deviation was 24. Find the number of students (i) exceeding the score 50

And (ii) between 30 and 54.

- Q.6 a. Find the inverse z- transform of $F(z) = \frac{1}{(z-3)(z-2)}$ if ROC is $2 < |z| < 3$. 06

- b Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. find the 06

diagonal form D and transforming matrix M.

- c 08

Use the dual simplex method to solve the L.P.P.

Maximize $z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 1;$

$$x_1 + x_2 \leq 7;$$

$$x_1 + 2x_2 \geq 10;$$

$$0x_1 + x_2 \leq 3;$$

$$x_1, x_2 \geq 0$$
