University of Mumbai

Question Bank for May / June (summer) 2022 End Semester Theory Examination
(2nd Half of A.Y. 2021-2022)

End Semester (Theory) Examinations Commencing from 17th May 2022 to 31st May 2022

Computer Engineering/Information Technology/Artificial Intelligence and Data Science/Artificial Intelligence and Machine Learning/Computer Science & Engineering (Data Science)

Curriculum Scheme: Rev 2019 'C' Scheme
Course Name: Engineering Mathematics III
Semester: III

	Multiple Choice Questions
1.	The Laplace transform of $\int_0^t \frac{1-e^{au}}{u}.du$ is
Option A:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option B:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option C:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
Option D:	$\frac{1}{s}\log\left(\frac{s-a}{s}\right)$
2	If $f(x) = \sqrt{(1-\cos x)}$, $0 < x < 2\pi$ then find a_0.
Option A:	$\frac{2\sqrt{2}}{\pi}$
Option B:	$\frac{\sqrt{2}}{\pi}$
Option C:	$\frac{\sqrt{2}}{3\pi}$
Option D:	$\frac{1}{\pi}$
3.	If $f(z) = u + iv$ is analytic then
Option A:	u is harmonic but v may or may not be harmonic.
Option B:	v is harmonic but u may or may not be harmonic.
Option C:	u and v both need not be harmonic.
Option D:	u and v both harmonic.
4.	If Var(X) = 4 then Var(3x+5) is
Option A:	12
Option B:	20
Option C:	26

Option D:	36
5.	If $f(x)$ is an even function in the interval $(-l, l)$ then in the Fourier series expansion of $f(x)$
Option A:	$a_n = 0, b_n = 0.$
Option B:	$a_n = 0, a_0 = 0.$
Option C:	$b_n = 0.$
Option D:	$a_0 = 0, b_n = 0.$
6	If $b_{yx} = 0.7764$, $b_{xy} = 1.2321$ then coefficient of correlation
Option A:	0.9781
Option B:	0.6291
Option C:	1.2307
Option D:	0.0023
7	Find the constants a, b, c, d if $f(z) = x^2 + 2axy + 2by^2 + i(2cx^2 + dxy + y^2)$
Option A:	$a = 1, b = -\frac{1}{2}, c = -\frac{1}{2}, d = 2.$
Option B:	$a = 0, b = -\frac{1}{2}, c = -\frac{1}{2}, d = 2.$
Option C:	$a = 1, b = -2, c = -\frac{1}{2}, d = 1.$
Option D:	$a = 3, b = -\frac{1}{2}, c = -\frac{1}{2}, d = 2.$
8	If X_1 has mean 4 and variance 9 and If X_2 has mean -2 and variance 4 and they are independent then $Var(2X_1 + X_2 - 3)$ is
Option A:	41
Option B:	40
Option C:	36
Option D:	37
9	Suppose two fair dice are thrown and sum of the numbers on dice is noted, what is the probability that the sum can be equal to 6, 7, 8 or 9.
Option A:	2/9
Option B:	5/9
Option C:	4/9
Option D:	7/9
10.	Let X denotes the demand in quintals and Y denotes the price in rupees per kg. Also if $\overline{X} = 68$, $\overline{Y} = 69$, $\sum\left(X - \overline{X}\right)^2 = 36$, $\sum\left(Y - \overline{Y}\right)^2 = 44$, $\sum\left(X - \overline{X}\right)\left(Y - \overline{Y}\right) = 24$ then the Karl Pearson's coefficient (r) of correlation is
Option A:	0.4030
Option B:	0.5030
Option C:	0.7030
Option D:	0.6030

11	Laplace transform of $e^{-5t}(t^2 + sin2t)$ *is*
Option A:	$\frac{2}{(s+5)^3} + \frac{2}{(s+5)^2 + 2^2}$
Option B:	$\frac{2}{(s-5)^3} + \frac{2}{(s-5)^2 + 4}$
Option C:	$\frac{3}{(s+5)^3} + \frac{s}{(s+5)^2 + 2^2}$
Option D:	$\frac{2}{(s+5)^2} + \frac{2}{(s+5)^2 - 2^2}$
12.	If $L\{F(t)\} = \frac{3s}{s^2+1}$, then $L\{F(2t)\}$ at s=1, is
Option A:	$\frac{3}{5}$
Option B:	$\frac{2}{5}$
Option C:	$-\frac{3}{5}$
Option D:	$\frac{7}{5}$
13.	Inverse Laplace transform of $\frac{1}{s^2+4}$ *is*
Option A:	$\int_0^t cos2udu$
Option B:	$\int_0^t sin2udu$
Option C:	$\int_0^t cos3udu$
Option D:	$\int_0^t cosudu$
14.	Inverse Laplace transform of f(s)= $\frac{6e^{-5s}}{(s+2)^4}$ *is*
Option A:	$f(t) = \begin{cases} 0 & 0 < t < 5 \\ e^{-2(t-5)}(t-5)^3 & t > 5 \end{cases}$
Option B:	$f(t) = \begin{cases} 0 & 0 < t < 5 \\ e^{-2(t-5)}(t-5)^4 & t > 5 \end{cases}$
Option C:	$f(t) = \begin{cases} 0 & t > 5 \\ e^{-2t}t^3 & t < 5 \end{cases}$
Option D:	$f(t) = \begin{cases} 0 & 0 < t < 5 \\ e^{-2t}t^5 & t > 5 \end{cases}$
15.	If $f(z) = u(x,y) + iv(x,y)$ is analytic then $f'(z)$*is equal to*
Option A:	$\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$
Option B:	$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

Option C:	$\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial x}$
Option D:	$\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$
16.	The value of 'm' so that $2x - x^2 + my^2$ is harmonic, is
Option A:	0
Option B:	-1
Option C:	1
Option D:	3
17.	The value of coefficient of correlation lies between
Option A:	0 to 1
Option B:	$-\infty \ to \ 1$
Option C:	0 to ∞
Option D:	-1 to 1
18.	The rank correlation coefficients of the following data is X: 23, 25, 27, 29, 31, 33 Y: 43, 45, 47, 49, 51, 53
Option A:	0
Option B:	-1
Option C:	1
Option D:	0.99
19.	Expansion of Fourier series of $f(x)=x$ in (-1, 1) is
Option A:	$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}(-1)^n \sin n\pi x$
Option B:	$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$
Option C:	$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x$
Option D:	$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x$
20	What would be the expectation of the number of failures preceding the first success in an infinite series of independent trials with the constant probability of success p and failure q
Option A:	$\frac{p}{q}$
Option B:	$\frac{q}{p}$
Option C:	$\frac{p+1}{q}$

Option D:	$\frac{p^2}{q^2}$
21.	Laplace Transform of t^3e^{-3t} is
Option A:	$\frac{7}{(s+4)^3}$
Option B:	$\frac{s}{(s+3)^3}$
Option C:	$\frac{6}{(s+3)^4}$
Option D:	$\frac{2}{(s+6)^3}$
22.	The value of $\int_0^\infty \frac{e^{-t}-e^{-4t}}{t}dt$ is
Option A:	log 4
Option B:	½ log 4
Option C:	log 1/4
Option D:	¼ log 2
23.	$L^{-1}\left[\frac{s-4}{s^2-4s+20}\right]$ is
Option A:	$e^{2t}\left(\cos\ 4t-\frac{1}{2}\sin 4t\right)$
Option B:	$e^{-2t}\left(\cos\ 4t+\frac{1}{2}\sin 4t\right)$
Option C:	$e^{2t}(\cos\ 4t-\sin 4t)$
Option D:	$e^{-2t}(\cos\ 4t-2\sin 4t)$
24.	$L^{-1}\left[\frac{1}{s(s+4)}\right]$ is
Option A:	$\frac{1}{4}(e^{-4t}-1)$
Option B:	$\frac{1}{4}(1-e^{-4t})$
Option C:	$(e^{-4t}-1)$
Option D:	$\frac{1}{4}(e^{-4t}+1)$
25.	If $f(x)=e^{-3x}$, $0<x<2\pi$ then a_0 is
Option A:	$\frac{1}{2\pi}(1-e^{-6\pi})$
Option B:	$(1-e^{-6\pi})$

Option C:	$\frac{1}{6\pi}(1+e^{6\pi})$
Option D:	$\frac{1}{6\pi}(1-e^{-6\pi})$
26.	Fourier coefficient b_2 in half range sine series for $f(x)=\frac{\pi}{6}\ in\ (0,\pi)$ is
Option A:	$\frac{\pi}{2}$
Option B:	π
Option C:	0
Option D:	$1/n$
27.	If $f(z)=ax^3-6xy^2+i(bx^2y-2y^3)$ is analytic then $a\ and\ b$ equals to
Option A:	$a=2, b=6$
Option B:	$a=2, b=0$
Option C:	$a=0, b=2$
Option D:	$a=-3, b=6$
28.	If one coefficient of regression is greater than one then other coefficient must be
Option A:	Zero
Option B:	Less than one
Option C:	Greater than one
Option D:	Not defined
29.	If X has the following probability distribution X: 0 1 2 $P(X=x)$: k 2k 5k Then the value of k is
Option A:	1/6
Option B:	0
Option C:	1/3
Option D:	1/8
30.	If X_1 has mean 4 and variance 9 and If X_2 has mean -2 and variance 4 and they are independent then $Var(2X_1+X_2-3)$ is
Option A:	41
Option B:	40
Option C:	36
Option D:	37

	Descriptive Questions
1	If $L\{\sin\sqrt{t}\}=\frac{\sqrt{\pi}}{2s\sqrt{s}}.e^{-1/(4s)}$, find $L\{\sin 2\sqrt{t}\}$
2	Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$
3	Find the Fourier series for $f(x)$ in $(0,2\pi)$ where $f(x)=\begin{cases} x, & 0<x\le\pi \\ 2\pi-x, & \pi\le x<2\pi \end{cases}$
4	If $v=3x^2y+6xy-y^3$, show that v is harmonic function and find the corresponding analytic function.
5	Calculate the value of rank correlation coefficient from the following data regarding marks of 6 students in Statistics and Mathematics in a test: *Marks*: *Statistics* : 40, 42, 45, 35, 36, 39 *Marks*: *Mathematics* : 46, 43, 44, 39, 40, 43
6	Three factories A, B, C produces 30% , 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A.
7	By using Laplace transform, prove that $\int_0^\infty e^{-t}.\frac{\sin^2 t}{t}dt=\frac{1}{4}\log 5$
8	Using convolution theorem, find the inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$
9	Obtain Fourier series for $f(x)=x+x^2;\ -1<x<1$
10	Find an analytic function $f(z)=u+iv$, where $u+v=e^x(\cos y+\sin y)$
11	State true or false with justification. "If two lines of regression are $x+3y-5=0$ and $4x+3y-8=0$, then the correlation coefficient is $+0.5$".
12	If the mean of the following distribution is 16. Find m,n and variance. X : 8, 12, 16, 20, 24 $P(X)$: 1/8 m n 1/4 1/12

<table>
<tr><td>13</td><td>Find the Laplace transform of $e^{-4t}\int_0^t u.\sin 3u\,du$</td></tr>
<tr><td>14</td><td>Find the inverse Laplace transform of $\tan^{-1}\left(\frac{a}{s}\right)$</td></tr>
<tr><td>15</td><td>Obtain half- range sine series for $f(x)$ where $f(x)=\begin{cases} x, & 0<x<(\pi/2) \\ \pi-x, & (\pi/2)<x<\pi \end{cases}$</td></tr>
<tr><td>16</td><td>Find the orthogonal trajectory of the family of curves given by
$2x-x^3+3xy^2=a$</td></tr>
<tr><td>17</td><td>Fit a straight line to the following data.
$(x,y)=(-1,-5),(1,1),(2,4),(3,7),(4,10)$ Estimate y when $x=7$</td></tr>
<tr><td>18</td><td>A random variable X has the following probability density function
$f(x)=\begin{cases} ke^{-kx}, & x>0,k>0 \\ 0, & elsewhere \end{cases}$
Find the moment generating function and hence, the mean and variance.</td></tr>
<tr><td>19</td><td>Find Laplace transform of $e^{-3t}t\sqrt{1-sin2t}$</td></tr>
<tr><td>20</td><td>Find inverse Laplace transforms of $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$</td></tr>
<tr><td>21</td><td>Expand Fourier Series for $f(x)=\frac{1}{2}(\pi-x)$ in $(0,2\pi)$.</td></tr>
<tr><td>22</td><td>Find constants a, b, c, d and e, if
$(ax^4+bx^2y^2+cy^4+dx^2-2y^2)+i(4x^3y-exy^3+4xy)$
is analytic.</td></tr>
<tr><td>23</td><td>Ten students got the following percentage of marks in mathematics and statistics
<table>
<tr><td>Maths</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr>
<tr><td>Stats</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr>
</table>
Calculate the coefficient of correlation.</td></tr>
<tr><td>24</td><td>A bolt is manufactured by three machines A, B and C . A turns out twice as many times as B, and machines B and C produce equal number of items . 3% of bolts produced by A and B are defective and 5% of bolts produced by C are defective . All bolts are put into one stock pile and one is chosen from this pile. What is the probability that it is defective?</td></tr>
<tr><td>25</td><td>By using Laplace transform, evaluate $\int_0^\infty \frac{sin2t+sin3t}{te^t}$</td></tr>
<tr><td>26</td><td>By using Convolution theorem, find inverse Laplace transform of</td></tr>
</table>

	$\frac{s}{(s^2+1)(s^2+4)}$
27	Expand Fourier Series for f(x) =1- x^2 in (-1, 1)
28	Find the analytic function $f(z) = u + iv$, in terms of z, if $v = \frac{\sinh 2y}{\cosh 2y + \cos 2x}$
29	Obtain the equations of the lines of regression for the following data. X: 65, 66, 67, 67, 68, 69, 70, 72 Y: 67, 68, 65, 68, 72, 72, 69, 71
30	A random variable X has the following probability distribution X: -2, -1, 0, 1, 2, 3 P: 0.1, K, 0.1, 2K, 0.2, 3K (i) Find the constant K. (ii) Find the mean and variance of X.
31	Find Laplace transform of $\int_0^t e^{-2u}\cos^2 u\,du$
32	Find Inverse Laplace transform of $\frac{1}{s}\log\sqrt{\left(\frac{s^2+9}{s^2+16}\right)}$
33	Find the half range cosine series for f (x) = $(x-1)^2$; 0< x < 1
34	Find the family of curves orthogonal to the family of curves $x^3y - xy^3 = c$
35	Fit a straight line of the form y=a+bx to the following data X: 1, 3, 5, 7, 8, 10 Y: 8, 12, 15, 17, 18, 20
36	A random variable x has probability density function $f(x) = \begin{cases} kx^2e^{-x} & x > 0, \quad k > 0 \\ 0 & Otherwise \end{cases}$ Find 'k' and hence find the mean and variance.
37	Find $L[\sin 3t \cos 2t\, cosht]$
38	Find $L^{-1}\left[\frac{s^2+10s+13}{(s-1)(s^2-5s+6)}\right]$
39	Find a Fourier series to represent the function $f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ \frac{1}{4}\pi x, & 0 < x < \pi \end{cases}$ Hence, deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ ....

40	If $v = e^x \sin y$ then prove that v is harmonic function. Also find the corresponding harmonic conjugate function.
41	Find Karl Pearson's coefficient of correlation between X and Y X: 3 6 4 5 7 Y: 2 4 5 3 6
42	Three urns contain 6 red, 4 black; 4 red, 6 black; 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn.
43	Find $L\left[t\sqrt{(1+sint)}\right]$
44	Find $L^{-1}\left[\frac{(s+3)^2}{(s^2+6s+18)^2}\right]$ using convolution theorem.
45	Find the Fourier series for $f(x) = \lvert x \rvert$ in $(-\pi, \pi)$
46	Determine the analytic function whose real part is $cosx\ coshy$
47	Two lines of regression are given by $5y - 8x + 17 = 0 \quad and \quad 2y - 5x + 14 = 0$ If $\sigma_y^2 = 16$, find i) mean value of x and y ii) σ_x^2 iii) coefficient of correlation between x and y
48	Find $L\left[\int_0^t ue^{-2u}cos^2 2u\ du\right]$
49	Find $L^{-1}\left[log\frac{(s^2+4)}{(s+2)^2}\right]$
50	Find half range cosine series for $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ x, & 1 \le x \le 2 \end{cases}$
51	Find orthogonal trajectories of the family of curves $x^2 - y^2 + x = c$
52	Fit a straight line to the following data X: 123 45 Y: 142740 5568
53	A random variable X has the following probability distribution X: 0 1 2 3 $P(X = x)$: 1/6 1/3 1/3 1/6 Find i) moment generating function about the origin ii) First four raw moments