

(Time: 2 hours)

Max Marks: 60

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

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Q1. Attempt any Five (Sub divisions) Questions from the following.

- a) Solve  $(x - 2e^y)dx + (y + x \sin x)dy = 0$ . 3
- b) Use Euler's method to find the approximate value of y correct up to four decimal places for  $x = 0.1$  given  $\frac{dy}{dx} = x - y^2$  when  $y(0) = 1$  and by considering  $h = 0.02$ . 3
- c) Evaluate  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz dy dx$  3
- d) Evaluate  $\int_0^\infty e^{-x^2} dx$  3
- e) Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 y dy dx$  3
- f) Solve  $(D^4 - 9D^2)y = 0$  3

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Q2.

- a) Solve  $\frac{dy}{dx} = x^2 + y$  where  $y(0.05) = 1.0513$  by Euler's modified formula for  $x = 0.1$  up to four decimal places and four approximations. 4
- b) Evaluate  $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \int_0^1 \frac{1}{\sqrt{1-x^4}} dx$ . 5
- c) Solve using method of variation of parameter  $(D^2 + 3D + 2)y = e^{e^x}$  6

Q3.

- a) Evaluate  $\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} dr d\theta$  4
- b) Solve  $(x^2 + y^2 + 2x)dx + 2y dy = 0$ . 5
- c) Apply Runge-Kutta method of forth order to find an approximate value of y at  $x = 0.2$  if  $\frac{dy}{dx} = x - y^2$ , given that  $y(0) = 1$  in two steps by taking  $h = 0.1$ . 6

Q4.

- a) Solve  $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^5}$  4
- b) If the density at any point on a circular lamina of radius 'a' is k-times the square of its distance from affixed point on the circumference, find mass. 5
- c) Solve the following differential equation  $\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^3}$ . 6

Q5.

- a) Prove that  $\int_0^1 \frac{x^a - 1}{\log x} dx = \log(1 + a)$ ,  $a \geq 0$ . 4
- b) Find the area bounded by  $y^2 = 4x$  and  $y = 2x - 4$ . 5
- c) Change the coordinate system and evaluate  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y \sqrt{x^2 + y^2} dy dx$  6

Q6.

- a) Solve the following differential equation  $(D^2 + 4D + 4)y = e^{2x} + \cos 2x$ . 4
- b) Change the order of integration  $\int_0^4 \int_{2-x}^{2+x} f(x, y) dy dx$  5
- c) Find the approximate value of  $\int_0^1 \frac{x^2}{1+x^3} dx$  using (i) Simpson's  $(1/3)^{rd}$  rule (ii) Trapezoidal rule and state which is the feasible method for the value of the integral. 6