Paper / Subject Code: N10411 / Applied Mathematics-I

FE (All branches) Sem-I (NEP-2020) June 2026

(Total Marks: 60)

03/06/2026

N.B.: **1) Question No. 1 is compulsory.**
2) Answer any three questions from Q.2 to Q.6.
3) Figures to the right indicate full marks.

1. Attempt any Five

(a) If $z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ and $\bar{z}$ is the conjugate of z, prove that $(z)^{10} + (\bar{z})^{10} = 0$. 3

(b) Prove that $\log\left(\frac{x+iy}{x-iy}\right) = 2i\tan^{-1}\left(\frac{y}{x}\right)$ 3

(c) Show that the matrix $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ 3

(d) Find the n^{th} derivative of $\frac{x}{(x+1)^4}$. 3

(e) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{x}\right)$ then find $\frac{\partial u/\partial x}{\partial u/\partial y}$. 3

(f) Using the Newton-Raphson Method, find a positive root of $3x - \cos x - 1 = 0$, correct to three decimal places. 3

2. (a) If α, β are the roots of the equation $z^2 \sin^2\theta - z\sin 2\theta + 1 = 0$, Then prove that 4

$$\alpha^2 - \beta^2 = 2\cos n\theta \operatorname{cosec}^n\theta.$$

(b) If $z = f(x, y), x = e^u + e^{-v}, y = e^{-u} - e^v$, Then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. 5

(c) Apply the Gauss-Seidel method to solve the equations up to three iterations. 6

$$10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$$

3. (a) Separate into real and imaginary parts $\tan^{-1}(x + iy)$. 4

(b) Find the value p, for which matrix $A = \begin{bmatrix} p & p & 2 \\ 2 & p & p \\ p & 2 & p \end{bmatrix}$ will have (i) rank 1, (ii) rank 2, (iii) rank 3. 5

(c) Discuss the maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 40$. 6

4. (a) If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$. 4

(b) If $u + iv = \operatorname{cosec}\left(\frac{\pi}{4} + ix\right)$, prove that $(u^2 + v^2)^2 = 2(u^2 - v^2)$. 5

(c) Find the root lying between 2 and 3 of the equation $x^3 - 4x - 9 = 0$ by the Regula Falsi method. 6

5. (a) Show that $z = f(x + at) + g(x - at)$ is solution of $a^2\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2}$ for all f and g.. 4

(b) If $x = \tan \log y$, prove that $(1 + x^2)y_{n+1} - (2nx - 1)y_n + n(n-1)y_{n-1} = 0$. 5

(c) Test for consistency the following equations and solve them if consistent 6

$$x - 2y + 3t = 2, \qquad 2x + y + z + t = 4, \qquad 4x - 3y + z + 7t = 8$$

6. (a) Find the n^{th} derivative of $y = \cos x \cos 2x \cos 3x$. 4

(b) Solve $x^5 = 1 + i$ and find the continued product of the roots. 5

(c) Prove that $\operatorname{Log}\left[\frac{(a-b) + i(a+b)}{(a+b) + i(a-b)}\right] = i\left(2n\pi + \tan^{-1}\frac{2ab}{a^2 - b^2}\right)$ 6

....

09004