Time: 3 hours **Max. Marks: 80**

N.B. : 1) Question no. 1 is compulsory.
2) Answer any 3 questions from remaining five questions.

1 **Answer any four questions**

(a) Define Discrete and Continuous random variables by giving examples. **5**

(b) Explain any two properties of Auto correlation Function. **5**

(c) Define SSS process. How it is different from WSS? **5**

(d) Define mathematical, statistical, and axiomatic definitions of probability. **5**

(e) Explain the central limit theorem. **5**

2. (a) For the following probability density function **10**

$$f_x(x) = \begin{cases} kx, & 0 \le x < 2 \\ k(4-x), & 2 \le x \le 4 \\ 0, & otherwise \end{cases}$$

(i) Find the value of k for which $f_x(x)$ is a valid pdf.
(ii) Find the mean and variance of X.
(iii) Find the CDF.

(b) Suppose X and Y are two random variables. Define Covariance and correlation of X and Y. When do you say that X and Y are **10**
i) Orthogonal
ii) Independent
iii) Uncorrelated
Are uncorrelated variables independent?

3. (a) Prove that for a linear time invariant system, if input is a WSS process, the output is also a WSS process. **10**

(b) The joint pdf of X and Y is given by, **10**

$$f_{x,y}(x,y) = e^{-(x+y)}; \ \ x > 0; y > 0$$

Find the pdf of $Z = \frac{x+y}{2}$

4. (a) A binary computer communication channel has the following error probabilities: P(R1 | S0) = 0.2, P(R0 | S1) = 0.06 where S0 = {'0' sent} S1 = {'1' sent} R0 = {'0' received} R1 = {'1' received} Suppose that 0 is sent with a probability of 0.8, find **10**
(a) The probability that '1' is received
(b) The probability that '1' was sent given that 1 is received
(c) The probability that '0' was sent given that '0' is received.

(b) Consider a random process $Y(t) = X(t)(cos\omega_0 t + \theta)$, where X(t) is a wide-sense stationary random process, θ is a random variable independent of X(t) and is distributed uniformly in (–π, π) and ω_0 is a constant. Prove that Y(t) is wide-sense stationary. **10**

5. (a) Let X be a random variable with PDF. Find Moment Generating Function of X and Var(X). **10**

$$f(x) = \frac{1}{2}\left(e^{-x/2}\right) \quad ; x > 0$$
$$= 0 \qquad ; \text{else}$$

(b) The density function of two random variables X and Y is; **10**

$$f_{x,y}(x,y) = \begin{cases} \frac{xy}{9}, & 0 < x < 2; 0 < y < 3 \\ 0, & elsewhere \end{cases}$$

(a) Show that X and Y are statistically independent.
(b) Show that X and Y are uncorrelated.

6. (a) Find the equation of the regression line from the following data. **10**

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

(b) State and prove Chebyshev inequality. **10**

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