Max Marks : 80 **Time : 3 Hours**

N.B. : 1) Question number 1 is **compulsory**
2) Answer **any three questions** from the remaining quuestions
3) Figures to the right indicate full marks

Q.1 a) Solve $x(x-1)\frac{dy}{dx}-(x-2)y=x^3(2x-1)$ (5)

b) Evaluate $\int_0^1 \sqrt{\sqrt{x}-x}\,dx$ (5)

c) Solve $(D^2+4)y=cos(2x)$ (5)

d) $\int_0^a \int_0^{\sqrt{a^2-x^2}}(x^2+y^2)\,dydx$ (5)

Q.2 a) Assuming the validity of differentiation under the integral sign, Prove that (6)

$$\int_0^1 \frac{x^a-x^b}{\log x}\,dx=\log\frac{a+1}{b+1}$$

b) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=\frac{e^{-2x}}{x^5}$ (6)

c) Change the order of integration for the integral $\int_0^a \int_{y2/a}^{y} f(x,y)dxdy$ (8)

Q.3 a) *Evaluate* $\int_0^3 e^{\sqrt{x}}dx$ *by Simpson's 3/8 th rule. Take h=0.25.* *(6)*

b) Apply Runge-Kutta method of fourth order to find an approximate value of y at y=0.2 if

$$\frac{dy}{dx}=x+y^2,$$

given that y=1 when x=0 in steps of h=0.1 (6)

c) Solve $(4xy+3y^2-x)dx+x(x+2y)dy=0$ (8)

Q.4 a) Solve $(D^3+D)y=\sin x$ (6)

b) Solve $(x^3y^3+xy)dy=dx$ (6)

c) Evaluate the integral $\int_0^2 \int_1^2 \int_0^{yz} xyzdxdydz.$ (8)

Q.5 a) Find $\int_0^1 \int_0^y xye^{-x^2}\,dxdy$ (6)

b) Evaluate $\int_0^2 y^4(8-y^3)^{\frac{-1}{3}}\,dy$ (6)

c) Solve (8)

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x},$$

Using the method of variation of parameters

Q.6 a) Solve $\frac{dy}{dx}\cosh x=2(\cosh x)^2\sinh x-y\sinh x$ (6)

b) a) Solve (6)

$\frac{dy}{dx}=2+\sqrt{xy}$ with $x_0=1.2, y_0=1.6403$ by Euler's modified formula for $x=1.4, x=1.6$

c) Evaluate $\iint ydxdy$ throughout the area bounded by x=0, $y=x^2$ and x+y=2. (8)