(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ 5

b) Solve $(D^2 - 4D + 4)y = e^{2x} + \cos 2x$ 5

c) Show that $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}}\,dx \, * \int_0^\infty y^4 e^{-y^6}\,dy = \frac{\pi}{9}$ 5

d) Change the order of the following integration 5

$$I = \int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y)\,dy\,dx$$

Q2.

a) Evaluate $I = \int\int\int \frac{dx\,dy\,dz}{(x^2+y^2+z^2)^{3/2}}$, over the volume V bounded by the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $(b > a)$ 6

b) Show that the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a[\sqrt{2} + \log(1 + \sqrt{2})]$. 6

c) Solve by using method of variation of parameters 8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

Q3.

a) Show that $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x}\,dx = \pi \sin^{-1}a \,, 0 \le a \le 1.$ Hence evaluate $\int_0^\pi \frac{\log(1+\cos x)}{\cos x}\,dx$ 6

b) Evaluate $I = \iint y^2 dxdy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$. 6

c) Evaluate $I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 yz\, dxdydz$ 8

Q4.

a) Solve $\cosh x \frac{dy}{dx} = 2\cosh^2 x \sinh x - y \sinh x$ 6

b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ 6

c) Show that $\int_0^{\infty} \frac{dx}{(e^x+e^{-x})^n} = \frac{1}{4}\beta\left(\frac{n}{2}, \frac{n}{2}\right)$, hence find the value of $\int_0^{\infty} sech^6 x\, dx$ 8

Q5.

a) Evaluate $I = \int_{-1}^{1} \int_0^{1-x} x^{1/3}\ y^{-1/2}\left((1-x-y)^{1/2}\right) dxdy$ 6

b) Find the area inside the circle $r = a$ and outside the cardioide $r = a(1 + \cos\theta)$ 6

c) Solve $xy\,(1 + xy^2)\frac{dy}{dx} = 1$ 8

Q6.

a) Solve $(D^2 + 2)y = x^2 e^{3x} + x \sin 3x$ 6

b) Solve $xe^x(dx - dy) + e^x dx + ye^y dy = 0$ 6

c) Change the order of integration and evaluate 8

$$I = \int_0^a \int_0^x \frac{dxdy}{(y+a)\sqrt{(a-x)(x-y)}}$$

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