(Time: 3 hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks

Q.1 a) Find $L(t + e^t + \cos t)^2$ [5]

Q.1 b) Find the Fourier series for $f(x) = x\ sinx$ in $(-\pi, \pi)$ [5]

Q.1 c) Find Karl Pearson's coefficients of correlation between X and Y from the following data [5]

X	100	200	300	400	500
Y	30	40	50	60	70

Q.1 d) If $f(z) = (x^3 + axy^2 + bxy) + i(3x^2y + cx^2 + y^2 + dy^3)$ is analytic, then find a, b, c, d [5]

Q.2 a) A random variable X has the following probability function [6]

X	1	2	3	4	5	6	7
P(X=x)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

Find i) k, ii) P(X≥ 4), iii) P(X < 5)

Q.2 b) Determine the analytic function whose real part is $u = e^x \cos y$ [6]

Q.2 c) Evaluate $\int_0^\infty e^{-t} \cosh t\ \cos 2t\ dt$. [8]

Q.3 a) Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$ [6]

Q.3 b) A continuous random variable X has the p.d.f. $f(x) = kx^2e^{-x},\ x \geq 0$ [6]

Find i) k, ii) $P(1 \leq x \leq 2)$

Q.3 c) Find $L^{-1}\left[\frac{s+29}{(s+4)(s^2+9)}\right]$ using partial fraction method [8]

Q.4 a) Find $L[f(t)],\ where\ f(t) = \cos t\ , 0 < t < \pi\ \ and\ f(t) = 0,\ t > \pi$ [6]

Q.4 b) Compute Spearman's rank correlation coefficient for the following data [6]

X	18	20	34	52	12
Y	39	23	35	18	46

Q.4 c) Obtain the Fourier series for [8]

$$f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 2 - \frac{\pi}{x}, & \pi \le x \le 2\pi \end{cases}$$

Q.5 a) Find $L^{-1}\left[\frac{4s+13}{s^2+8s+13}\right]$ [6]

Q.5 b) Find $L[(1 + sin2t)^2]$ [6]

Q.5 c) Find the line of regression of Y on X for the following data [8]

X	5	6	7	8	9	10	11
Y	11	14	14	15	12	17	16

Q.6 a) Find mean and variance for the following distribution [6]

X	8	12	16	20	24
P(X = x)	1/8	1/6	3/8	1/4	1/12

Q.6 b) Find $i)\ L^{-1}[cot^{-1}2s]$ $ii)\ L^{-1}\left[\log\left(1 + \frac{4}{s^2}\right)\right]$ [6]

Q.6 c) Prove that the function $f(z) = e^{2z}$ is analytic. Also, find its derivative. [8]

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