

Time:3Hours

Max. Marks: 80

**N.B.**

- 1) **Q.1 is compulsory.**
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

**Q.1 Solve any four of the following questions.**

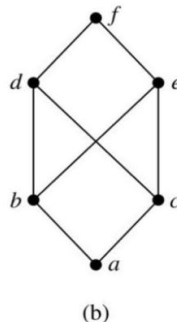
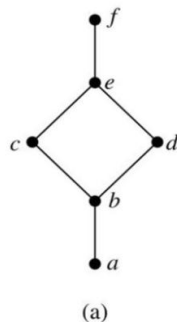
- a) Check if  $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$  is a tautology? 5
- b) Draw the Hasse diagram for  $[\{2,4,5,8,10,12,20,25\}, /]$ . Is it a Poset? 5
- c) Define Eulerian and Hamiltonian Graph. Give examples of following type of graph 5
  - i) Eulerian but not Hamiltonian
  - ii) Not Eulerian but Hamiltonian
- d) Explain types of Quantifiers . Represent the following sentences using Quantifiers 5
  - i) All hardworking students are clever.
  - ii) There is a student who can speak Hindi but does not know Marathi
- e) State the Pigeonhole principle and prove that in any set of 29 persons at least five persons must have been born on the same day of the week 5

**Q.2**

- a) Show that the set of all positive rational numbers forms an abelian group under the composition \* defined by  $a*b=(ab)/2$  10
- a) What is a transitive closure? Explain Warshall's algorithm for finding transitive closure with an example. 10

**Q.3**

- a) By using mathematical induction, prove that the given equation is true for all positive integers. 6  
 $1 \times 2 + 3 \times 4 + 5 \times 6 + \dots + (2n - 1) \times 2n = n(n+1)(4n-1)/3$
- b) Define Lattice? Which of the following is lattice? 8



- c) Determine the sequence of which recurrence relation is  $a_n = 2a_{n-1} - a_{n-2}$  with initial conditions  $a_1= 1.5, a_2= 3$ . 6

**Q.4**

- a) Let  $A = \{1, 3, 6, 9, 15, 18, 21\}$  &  $R$  be the relation of divisibility. 8  
 i) Write the pairs in a relation set  $R$ .  
 ii) Construct the Hasse diagram.  
 iii) What are the Maximal and Minimal elements?  
 iv) Is this poset a distributive lattice? Justify your answer.

b)

Let  $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  be a parity check matrix. Determine  $(3, 6)$  group code  $e_H : B^3 \rightarrow B^6$  6

c) Write a short note on Types of Graphs. 6

**Q.5**

a) Let  $(Z, *)$  be an algebraic structure, where  $Z$  is set of integers and the operation  $*$  is defined by  $a*b = \text{maximum of } (a,b)$ . Is  $(Z, *)$  a Semigroup? Is  $(z, *)$  a Monoid? Justify your answer. 8

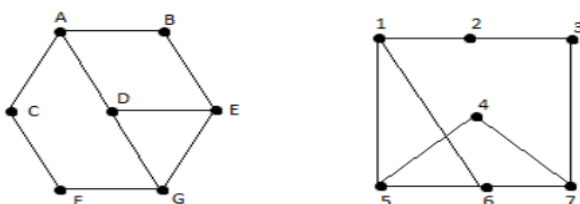
b) Define the term Surjective function. Let  $E$  be the set of all even numbers then  $f: N \rightarrow E: f(x) = 2x$ , check if it is surjective, bijective? Justify your answer. 4

c) Give the examples of relation  $R$  on  $S = \{a, b, c, d\}$  having stated property. 8

- i)  $R$  is an equivalence relation.
- ii)  $R$  is symmetric but not transitive
- iii)  $R$  is both symmetric and antisymmetric
- iv)  $R$  is neither symmetric nor antisymmetric.

**Q.6**

a) Define Isomorphic graphs and check whether the following graphs are Isomorphic ? 8



b) In a group  $(G, *)$ , Prove that the inverse of any element is unique and identity element is also unique. 6

c) Define Relation. Let 6  
 $f: R \rightarrow R$  is defined as  $f(x) = x^2$   
 $g: R \rightarrow R$  is defined as  $g(x) = 3x^2 + 1$   
 $h: R \rightarrow R$  is defined as  $h(x) = 9x - 2$   
 find  $(hof)og, go(foh)$ .

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