Duration: 3hrs **[Max Marks: 80]**

N.B. : (1) Question No 1 is Compulsory.
(2) Attempt any three questions out of the remaining five.
(3) All questions carry equal marks.
(4) Assume suitable data, if required and state it clearly.

1 Attempt any FOUR [20]

a If R be a relation in the set of integers z defined by

$R = \{ (x, y): x \in z, y \in z, x - y \text{ is divisible by } 3 \}$

Show that the relation R is an equivalence relation.

b Prove using Mathematical Induction that
$P(n) = 1.1! + 2.2! + \cdots . + 3.3! = (n + 1)! - 1$

c Design an FSM in which input is valid if it ends in "1011" over ∑= {0,1}

d Design NFA for the regular expression

$$R = (0(0 + 1)^* 10)$$

e Differentiate between DFA and NFA

2 a Define Poset. Draw Hasse diagram which represents the partial order relation. [10]

$R = \{(a, b) | a \text{ divides } b \}$ on {1,2,3,4,6,8,12}

b Convert the following NFA to DFA: [10]

Q/∑	0	1
→ p	p,q	p
q	r,s	t
r	p ,r	t
s^*	ϕ	ϕ
t^*	ϕ	ϕ

3 a Simplify the following CFG: [10]

$$S \to aAa | bBb | BB$$
$$A \to C$$
$$B \to A | S$$
$$C \to S | \in$$

b Write a short note on Warshall's algorithm. [10]
Let $A = \{a1, a2, a3\}$ and R be a relation on A whose matrix is:

$Mr = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find transitive closure of R using Warshall's algorithm.

4 a Design PDA to check odd palindrome over Σ= {0,1} [10]

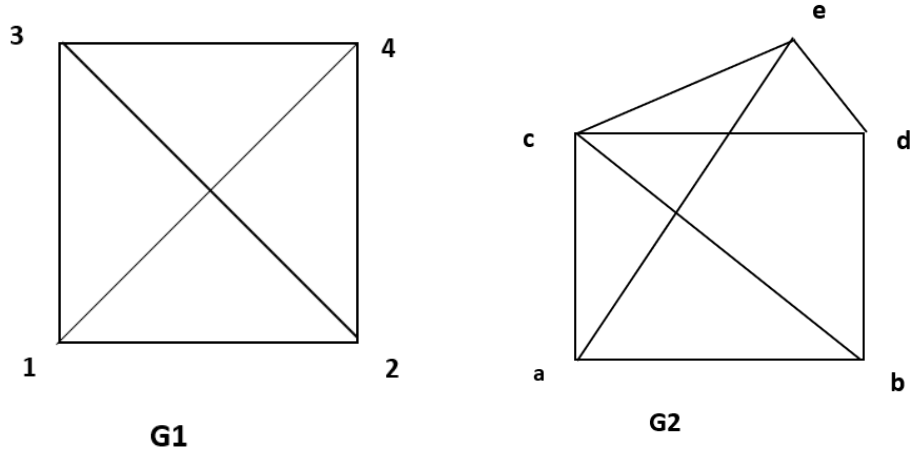
b Give and explain formal definition of pumping lemma for regular language and prove that the following language is not regular. [10]

$$L = \{ a^m b^{m-1} \mid m > 0 \}$$

5 a Design Moore machine for the following: If input ends in '101'then output should be 'A', if input ends in '101' output should be 'B', otherwise output should be 'C' and convert it into Mealy machine. [10]

b Design a finite automaton to check divisibility by 3 to binary number. [10]

6 a Determine if the following graphs G1 and G2 are isomorphic or not. [10]



b Define injective, surjective and bijective functions. If $f: R \to R$ and $g: R \to R$ are defined by the formulas: $f(x) = x + 2$ and $g(x) = x^2$. Find [10]

1. $f.g.f$
2. $g.f.g$