

(Time: 3 hours )

Max. Marks: 80

- N.B. (1) Question No. 1 is compulsory.  
 (2) Answer any three questions from Q.2 to Q.6.  
 (3) Use of Statistical Tables permitted.  
 (4) Figures to the right indicate full marks

Q1.

- (a) Find the Laplace transform of  $t \sqrt{1 + \sin t}$  [ 5 ]  
 (b) Find the constants a, b, c, d, e if [ 5 ]  
 $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$  is analytic.  
 (c) Calculate the Spearman's rank correlation coefficient R [ 5 ]  
 X : 85, 74, 85, 50, 65, 78, 74, 60, 74, 90  
 Y : 78, 91, 78, 58, 60, 72, 80, 55, 68, 70  
 (d) Find inverse Laplace transform of  $\tan^{-1} \left( \frac{s+a}{b} \right)$ . [ 5 ]

Q2.

- (a) Find the Laplace transform of  $e^{-4t} \int_0^t u \sin 3u du$  [ 6 ]  
 (b) find the value of k if the function  $f(x) = kx^2(1-x^3)$ ,  $0 \leq x \leq 1$ .  
 $F(x) = 0$  otherwise  
 Is a probability density function. find mean and variance. [ 6 ]

- (c) Obtain the Fourier series to represent  $f(x) = x^2$  in  $(0, 2\pi)$

Hence show that  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$  [ 8 ]

Q3.

- (a) Find the analytic function  $f(z) = u + iv$  such that [ 6 ]

$$u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x} .$$

(b) Using convolution theorem Find inverse Laplace transform of  $\frac{s^2}{(s^2+9)(s^2+4)}$ . [ 6 ]

(c) Fit a second-degree parabolic curve to the following data

Year (x)	: 1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	: 12	14	26	42	40	50	52	53.

[ 8 ]

Q4.

(a) Obtain the Fourier series to represent  $f(x) = 9 - x^2$  in  $(-3, 3)$ . [ 6 ]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X: 78, 36, 98, 25, 75, 82, 90, 62, 65, 39.

Y: 84, 51, 91, 60, 68, 62, 86, 58, 53, 47. [ 6 ]

(c) Prove that  $\int_0^\infty e^{-t} \frac{\sin 2t + \sin 3t}{t} dt = \frac{3\pi}{4}$ . [ 8 ]

Q5.

(a) Find the orthogonal trajectories of the family of curves  $3x^2y + 2x^2 - y^3 - 2y^2 = c$ . [ 6 ]

(b) If X denotes the outcome when a fair die is tossed, find Moment generating function Of X and hence find the mean and variance of X. [ 6 ]

(c) Obtain the half range cosine series of  $f(x) = x(\pi - x)$  in  $(0, \pi)$

Hence show that  $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$  [ 8 ]

Q6.(a) Find inverse Laplace transform of  $\frac{s+29}{(s+4)(s^2+9)}$ . [ 6 ]

(b) The probability density function of a random variable X is

X	: 0	1	2	3	4	5	6
P ( X = x )	: k	3k	5k	7k	9k	11k	13k

Find k ,  $p(X < 4)$  ,  $P(3 < X \leq 6)$  . [ 6 ]

(c) Verify Laplace equation for  $u = \left(r + \frac{a^2}{r}\right) \cos \theta$ . also find v and  $f(z)$  . [ 8 ]

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