(Time: 3 hours) **[Total marks: 80]**

Note: - (1) Question No. 1 is compulsory.

(2) Answer any three question from Q 2 to Q 6.

(3)Figures to the right indicate full marks.

1 (a) Find the Laplace Transform of $e^{-t}\sin 2t\cos 3t$. **05**

1 (b) Construct an analytic function whose real part is $y^3 - 3x^2y$ **05**

1 (c) Find Eigen values of $A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -3 \end{bmatrix}$. **05**

1 (d) Find Fourier series for $(x) = x^2 \quad in \quad 0 < x < 2\pi$. **05**

2 (a) If $\bar{F} = xye^{2z}i + xy^2coszj + x^2$cosxyk find div$\bar{F}$ and curl $\bar{F}$ **06**

2 (b) Find Fourier series of $f(x) = x^3$, $-\pi < x < \pi$. **06**

2 (c) Find Inverse Laplace Transform of (i) $\frac{2s+3}{s^2+2s+2}$ (ii) $\frac{s+2}{s(s+3)}$. **08**

3 (a) Find Eigen Values and Eigen Vector of the following matrix **06**

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

3 (b) Determine the Constants a, b, c, d if

$f(z) = x^2 + 2axy + by^2 + i\,(dx^2 + cxy + y^2)$ is analytic **06**

3 (c) Find Fourier series for $f(x) = \begin{cases} 1 + {}^{2x}/_{\pi}, & -\pi < x < 0 \\ 1 - {}^{2x}/_{\pi}, & 0 < x < \pi \end{cases}$ and hence deduce that **08**

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots$$

4 (a) Prove that $\bar{F} = (ysinz - sinx)i + (xsinz + 2yz)j + (xycosz + y^2)k$ is solenoidal and irrotational . **06**

4 (b) Evaluate $\int_0^\infty e^{2t} t \cos t \ dt$. **06**

4 (c) Show that the matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$ diagonalizable and find transforming matrix and Diagonal matrix. **08**

5 (a) Find the inverse Laplace Transform of $\frac{s+4}{(s+1)(s+2)(s+3)}$ by using Partial fraction method. **06**

5 (b) Construct an analytic function $f(z) = u + iv$, where **06**

$v = (x - y)(x^2 + 4xy + y^2)$.

5 (c) i) Show that $\bar{F} = (2xyz^2)i + (x^2z^2 + zcosyz)j + (2x^2yz + ycosyz)k$ is a conservative field. **04**

ii) If $\bar{F} = (x + 3y)i + (y - 2z)j + (az + x)k$ is solenoidal, find the value of a **04**

6 (a) Find Eigen Values and Eigen Vector of the following matrix **06**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

6 (b) Find inverse Laplace transform

i) $\frac{1}{s^2+3s+5}$ ii) $log\left[\frac{s^2+4}{s+4}\right]$ **06**

6 (c) Evaluate $\int_0^\infty e^{-2t}\left(\int_0^t e^{-u} u \ sin\ 2u \ du\right) dt$ **08**
