

Time: 3 hrs.

Max. Marks: 80

- N.B. :** 1. Q1 is compulsory
 2. Attempt any three questions from Q2 to Q6.
 3. Figures to the right indicate full marks.

Q1. (a) Evaluate the integral $\int_C \frac{z^2}{(z-i)(z+2)^2} dz$, $C: |z-i|=1$. 5

(b) A r.v. X has the distribution 5

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

Find i) k ii) evaluate P(X < 6)

(c) Find the usual inner product between the two vectors (2,1,-3) 5

and (-1,1,2). Find the norm of each vectors and verify the Cauchy

Schwarz inequality.

(d) The given data indicates weight x and heights y of 1000 men. $\bar{x} = 150$ lbs, 5

$\bar{y} = 68$ inches, $\sigma_x = 20$ lbs, $\sigma_y = 2.5$ inches, $r = 0.6$. John weighs 200 lbs.

Find the line of regression of y on x and estimate the height of John.

Q2. (a) Find the Extremal of $\int_{x_1}^{x_2} \sqrt{1+(y')^2} dx$. 6

(b) Find the Laurent series expansion of $\frac{z+2}{z^2-1}$ convergent in the 6

domain $|z| > 1$.

(c) Reduce the quadratic form $x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + x_1x_3 - 2x_2x_3$ 8

to diagonal form by congruent transformation. Obtain the transformation applied in the reduction and Find the rank, index and class value.

Q3. (a) Find the Extremal of $\int_0^1 y y' + (y'')^2 dx$, 6

$$y(0) = 0, y'(0) = 1, y(1) = 2, y'(1) = 4$$

(b) From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and found to be white. Find the probability that out of four balls transferred 3 are white and 1 is black. 6

(c) Find a singular value decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. 8

Q4. (a) Evaluate the integral $\int_C \frac{\sin^2 z}{z^3} dz$, $C: |z|=1$, using Cauchy integral formula. 6

(b) Using Gram Schmidt method, find an orthogonal set of vectors corresponding to $(1,1,0,1)$, $(-1,0,1,0)$, $(0,0,1,-1)$. 6

(c) After correcting 50 pages of the proof of a book, the proof reader finds that there are on the average 2 errors per 5 pages. How many pages would one expect to find with 0, 1, 2, 3 errors in 1000 pages of the first print of the book. 8

Q5. (a) Evaluate the Integral $\int_C \bar{z} dz$ along a straight line 6

$$\text{from } z = 0 \text{ to } z = 4 + 2i.$$

(b) Find the rank correlation coefficient for the following data. 6

$$x : 10 \quad 12 \quad 18 \quad 16 \quad 15 \quad 40$$

$$y : 12 \quad 18 \quad 20 \quad 15 \quad 50 \quad 25$$

(c) Using Rayleigh-Ritz method, find an approximate solution for the 8

$$\text{Extremal of } \int_0^1 (y')^2 - 4y^2 + 2x^2 y dx, \quad y(0) = 0, y(1) = 0$$

Q6. (a) If $f(x) = \begin{cases} \frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$ is a pdf of a random variable X, then 6

find $E(X)$, $\text{var}(X)$, $\text{var}(3X)$.

(b) Let $W_1 = \{(x, y) \mid x, y \text{ are real numbers, } y = mx\}$ and 6

$W_2 = \{(x, y) \mid x, y \text{ are real numbers, } xy \geq 0\}$. Show that

W_1 is a subspace and W_2 is not a subspace of two dimensional space.

(c) A Chemical Engineer is investigating the effect of process operating 8

temperature x on product yield y . The study results in the following data

x : 100 110 120 130 140 150 160 170 180 190

y : 45 51 54 61 66 70 74 78 85 89

Find the equation of the least square lines which will enable us to predict

(i) yield on the basis of temperature (ii) temperature on the basis of yield.