

(Time : 3 hours)

[Total marks: 80]

**Note :- (1) Question No. 1 is compulsory.****(2) Answer any three question from Q 2 to Q 6.****(3) Figures to the right indicate full marks .**1(a) Find the Laplace Transform of  $e^t \sin 2t \sin 3t$  . 051(b) Construct an analytic function whose imaginary part is  $v = \cos x \sinh y$  . 051(c) Find Eigen values of  $A^2 - 2A + I$  where  $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ . 051(d) Find the Fourier Series Expansion  $f(x) = x$ , where  $x \in (-\pi, \pi)$  052(a) Find the direction derivative of  $\phi(x, y, z) = \sin(xy) + e^{3xz}$  in the direction of the vector  $v = i - 2j + 2k$  at the point  $P = \left(1, \frac{\pi}{4}, 1\right)$  062(b) Find Fourier series of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$  . 062(c) Find Inverse Laplace Transform of (i)  $\frac{2s+3}{s^2+2s+2}$  (ii)  $\frac{s+2}{s^2(s+3)}$ . 08

3(a) Find Eigen Values and Eigen Vector of the following matrix 06

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

3(b) Find orthogonal trajectory of family of curve  $3x^2y - y^3 = c$  . 063(c) Find the Fourier Series for  $f(x)$  in  $(0, 2\pi)$  where

$$f(x) = \begin{cases} x & , 0 < x \leq \pi \\ 2\pi - x & , \pi \leq x < 2\pi \end{cases}$$

Hence deduce that

$$\sum_{n \in \text{Odd natural numbers}} \frac{1}{n^4} = \frac{\pi^4}{96} \quad 08$$

4(a) Prove that is  $\vec{F} = \frac{xi+yj}{x^2+y^2}$  is both solenoidal and irrotational . 06

4(b) Evaluate  $\int_0^\infty e^{-2t} t \cos t dt$ , 06

4(c) Show that the matrix

$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  diagonalizable and find transforming matrix and Diagonal matrix . 08

5(a) Find the inverse Laplace Transform of  $\frac{(s+2)^2}{(s^2+4s+8)^2}$  by using convolution theorem. 06

5(b) Construct an analytic function  $f(z) = u + iv$ , where 06

$u - v = (x - y)(x^2 + 4xy + y^2)$  .

5(c) Evaluate by using Green's theorem

$\int_C (e^{x^2} - xy)dx + (y^2 - ax)dy$ , where C is the circle  $x^2 + y^2 = a^2$ . 08

6(a) By using CHT theorem and find  $A^{-1}$  and  $A^4$  . where  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  06

6(b) Obtain half range sine series in  $(0, \pi)$  for  $f(x) = x(\pi - x)$ ,

Hence show that

$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$  06

6(c) Evaluate  $\int_0^\infty e^{-2t} \left( \int_0^t \frac{e^{-u} \sin 2u}{u} du \right) dt$  08