

(Time: 3 hours)

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks.

Q1 (a) Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ given that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-(1/4s)}$ [5]

(b) Calculate Spearman's rank correlation coefficient for the following data: [5]

X	32	55	49	60	43	37	43	49	10	20
Y	40	30	70	20	30	50	72	60	45	25

(c) Find inverse Laplace transform of $\frac{2s-1}{s^2+8s+29}$ [5]

(d) If $f(z) = qx^2y + 2x^2 + ry^3 - 2y^2 - i(px^3 - 4xy - 3xy^2)$ is analytic, find the values of p, q, and r [5]

Q2 (a) Find Laplace transform of $e^{3t} f(t)$ where $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$ [6]

(b) Two unbiased dice are thrown. If X represents sum of the numbers on the two dice. Write probability distribution of the random variable X and find mean, standard deviation, and $P(|X-7| \geq 3)$ [6]

(c) Obtain Fourier series for $f(x) = x \sin x$ in the interval $0 \leq x \leq 2\pi$. [8]

Q3 (a) Using Milne-Thompson's method construct an analytic function $f(z) = u + iv$ in terms of z where $u + v = e^x(\cos y + \sin y) + \frac{x-y}{x^2+y^2}$ [6]

(b) Using convolution theorem find the inverse Laplace transform of $\frac{(s+3)^2}{(s^2+6s+5)^2}$ [6]

(c) Fit a parabola $y = a + bx + cx^2$ to the following data and estimate y when $x=10$ [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Q4 (a) Find Laplace transform of $e^{-(1/2)t} t f(3t)$ if $L\{f(t)\} = \frac{1}{s\sqrt{s+1}}$ [6]

- (b) Find half range sine series for $f(x) = x - x^2$, $0 < x < 1$. [6]

Hence deduce that
$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$$

- (c) Given regression lines $6y = 5x + 90$, $15x = 8y + 130$, $\sigma_x^2 = 16$.
Find i) \bar{x} and \bar{y} , ii) r , iii) σ_y^2 and iv) angle between the regression lines [8]

- Q5 (a) Can the function $u = r + \frac{a^2}{r} \cos\theta$ be considered as real or imaginary part of an analytic function? If yes, find the corresponding analytic function. [6]

- (b) An unbiased coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails, find moment generating function of X and hence obtain the first moment about origin and the second moment about mean. [6]

- (c) Evaluate $\int_0^\infty e^{-2t} \cos t \int_0^t u^2 \sinh u \cosh u \, du \, dt$ [8]

- Q6 (a) Find inverse Laplace transform of $\frac{1}{(s-2)^4(s+3)}$ using method of partial fractions. [6]

- (b) If a continuous random variable X has the following probability density function [6]

$$f(x) = \begin{cases} k e^{-\frac{x}{4}}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases} \quad \text{find } k, \text{ mean and variance.}$$

- (c) Find half range cosine series for $f(x) = x$, $0 < x < 2$. [8]

Hence deduce that i)
$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = \frac{\pi^4}{96}$$

ii)
$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$$
