

Time: 3 Hours

Total Marks:80

Instructions:

- 1) Question 1 is compulsory
- 2) Attempt any three from the remaining questions.

1-a) Prove that the matrix  $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$  is unitary. (5 Marks)

1-b) State Euler's theorem on homogeneous function of two variables and evaluate  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  where,  $u = \frac{x+y}{x^2+y^2}$ . (5 Marks)

1-c) Separate into real and imaginary part of  $\cos^{-1} \left( \frac{3i}{4} \right)$ . (5 Marks)

1-d) If  $y = 2^x \sin^2 x \cos x$  find  $y_n$ . (5 Marks)

2-a) Show that  $\frac{\sin 5\theta}{\sin \theta} = 16\cos^4 \theta - 12\cos^2 \theta + 1$  (6 Marks)

2-b) If  $u = \tan^{-1} \left( \frac{x^2+y^2}{x-y} \right)$  P.T  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$  (6 Marks)

2-c) Test for consistency the following system & solve them if consistent. (8 Marks)

$$x_1 - 2x_2 + x_3 - x_4 = 2$$

$$x_1 + 2x_2 + 2x_4 = 1$$

$$4x_2 - x_3 + 3x_4 = -1$$

3-a) Show that minimum value of  $u = xy + a^3 \left( \frac{1}{x} + \frac{1}{y} \right)$  is  $3a^2$ . (6 Marks)

3-b) Using Newton-Raphson method find the root of equation  $2x^3 - 3x + 4 = 0$  lying between -2 and -1 correct to four places of decimals. (6 Marks)

3-c) If  $y^{1/m} + y^{-1/m} = 2x$  prove that  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ . (8 Marks)

4-a) Solve  $x^5 = 1 + i$  and find the continued product of the roots. (6 Marks)

4-b) Apply Gauss elimination method to solve the equations  $x+3y-2z=5$ ,  $2x+y-3z=1$ ,  $3x+2y-z=6$ . (6 Marks)

4-c) For what value of  $\lambda$  the equations  $x + 2y + z = 3$ ,  $x + y + z = \lambda$ ,  $3x + y + 3z = \lambda^2$  have a solution and solve them completely in each case. (8 Marks)

5-a) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x}{3} \right)^{1/x}$ . (6 Marks)

5-b) If  $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ , then show that  $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$ . (6 Marks)

5-c) Prove that  $\log \left[ \frac{\sin x + iy}{\sin x - iy} \right] = 2i \tan^{-1}(\cot x \tanh y)$  (8 Marks)

6-a) Find the  $n^{\text{th}}$  derivative of  $\frac{x}{(x-1)(x-2)(x-3)}$  (6 Marks)

6-b) Reduce the following matrix to its normal form and hence find its rank.

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix} \quad (6 \text{ Marks})$$

6-c) i) Express  $(2x^3 + 3x^2 - 8x + 7)$  in terms of  $(x - 2)$  using Taylor's theorem.

ii) Prove that  $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots$  (8 Marks)