# University of Mumbai

## Examinations Summer 2022

**Program No: 1T01831**

**Examination: F.E. (Sem I) (ALL BRANCHES) (Rev 2019 'C'-Scheme)**

**Subject (Paper Code): 58651 // Engineering Mathematics - I**

**Time: 2 hour 30 minutes** **Max. Marks: 80**

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| **Q I.** | **Choose the correct option for following questions. All the Questions are compulsory and carry equal marks.** |
| 1. | The value of $\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)^{10}+\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{10}$ is equal to |
| Option A: | $\frac{\pi}{2}$ |
| Option B: | 0 |
| Option C: | $\frac{\pi}{3}$ |
| Option D: | $\frac{\pi}{4}$ |
| | |
| 2. | What is the value of $\log(i)$ |
| Option A: | $i\,\frac{\pi}{2}$ |
| Option B: | 0 |
| Option C: | $-2$ |
| Option D: | $-i\,\frac{\pi}{2}$ |
| | |
| 3. | $If\ u=\log(tanx+tany)$ then the value of $sin2x\frac{\partial u}{\partial x}+sin2y\frac{\partial u}{\partial y}$ is |
| Option A: | 2 |
| Option B: | -1 |
| Option C: | 0 |
| Option D: | - 2 |
| | |
| 4. | All the stationary points of the function $x^3+3xy^2-15x^2-15y^2+72x$ are |
| Option A: | $(6,0),\quad (4,0),\quad (5,1),\quad (5,-1)$ |
| Option B: | $(6,4),\quad (4,0),\quad (5,0),\quad (5,1)$ |
| Option C: | $(6,0),\quad (0,0),\quad (5,1),\quad (5,-1)$ |
| Option D: | $(0,0),\quad (4,0),\quad (5,1),\quad (5,-2)$ |
| | |
| 5. | If $A=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&3\end{bmatrix}$, then rank of A is |
| Option A: | 2 |
| Option B: | 3 |
| Option C: | 1 |
| Option D: | 0 |

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| 6. | The modulus and principal value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$ is |
| Option A: | $\frac{1}{4}(cos\frac{\pi}{6}+isin\frac{\pi}{6})$ |
| Option B: | $4(cos\frac{\pi}{3}+isin\frac{\pi}{3})$ |
| Option C: | $\frac{1}{4}(cos\frac{\pi}{3}+isin\frac{\pi}{3})$ |
| Option D: | $4(cos\frac{\pi}{6}+isin\frac{\pi}{6})$ |
| | |
| 7. | The real part of $\cos^{-1}\left(\frac{3i}{4}\right)$ is |
| Option A: | π |
| Option B: | 2 π |
| Option C: | - π |
| Option D: | π/2 |
| | |
| 8. | $If\ u=\frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}\ \ then\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\ is$ |
| Option A: | $\frac{u}{2}$ |
| Option B: | $\frac{-u}{2}$ |
| Option C: | $2u$ |
| Option D: | $\sqrt{2}\,u$ |
| | |
| 9. | Stationary point is a point where $f(x,y)$ has |
| Option A: | $\frac{\partial f}{\partial x}=0$ |
| Option B: | $\frac{\partial f}{\partial y}=0$ |
| Option C: | $\frac{\partial f}{\partial x}=0,\ \frac{\partial f}{\partial y}=0$ |
| Option D: | $\frac{\partial f}{\partial x}<0\ \frac{\partial f}{\partial x}>0$ |
| | |
| 10. | For non-singular matrices P and Q, PAQ is in the normal form of a matrix A, then $A^{-1}$ can be found by |
| Option A: | $A^{-1} = Q^{-1} P$ |
| Option B: | $A^{-1} = P Q^{-1}$ |
| Option C: | $A^{-1} = QP$ |
| Option D: | $A^{-1} = Q P^{-1}$ |

| Q II. (20 Marks) | Solve any Four out of Six. 5 marks each |
|---|---|
| A | Prove that: $\frac{\sin 6\theta}{\sin 2\theta} = 16cos^2\theta - 16cos^2\theta + 3$ |
| B | Considering only principal values separate into real and imaginary parts $i^{\log(1+i)}$. |
| C | If z = $tan^{-1}\left(\frac{y}{x}\right)$, find the value of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$. |
| D | Find the extreme value of the function $xy(3 - x - y)$. |
| E | Express the matrix $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ as a sum of Hermitian and skew Hermitian matrix. |
| F | If $y = a\cos(logx) + b\,sin\,(logx)$, then show that $x^2 y_{n+2} + (2n+1)x\,y_{n+1} + (n^2+1)y_n = 0.$ |

| Q III. (20 Marks) | Solve any Four out of Six. 5 marks each |
|---|---|
| A | Find all the values of $(1+i)^{\frac{1}{3}}$ and show that their continued product is $(1+i)$. |
| B | Separate into real and imaginary parts $tan^{-1}(\alpha + i\beta)$ |
| C | If $u = sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u\,\cos 2u}{4cos^3 u}$. |
| D | Divide 24 into 3 parts such that the continued product of the first, square of second and cube of the third is maximum using Lagrange's method. |
| E | Find a, b, c if A is orthogonal matrix where A $=\frac{1}{3}\begin{bmatrix} a & b & c \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$. Hence find inverse of A. |
| F | Investigate for what values of $\lambda$ and $\mu$ the system of equations $x + y + z = 6$ ; $x + 2y + 3z = 10$ ; $x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) an infinite no. of solutions. |

| Q IV. (20 Marks) | Solve any Four out of Six. 5 marks each |
|---|---|
| A | Prove that $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1}\cos\frac{n\pi}{3}$. |
| B | Prove that $\sinh^{-1}(\tan\theta) = log\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$ |
| C | If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$ |
| D | Find n[th] derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$. |

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| E | Find non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is reduced to normal form. Also find its rank. |
| F | Using De Moivre's theorem prove that $$cos^6\theta - sin^6\theta = \frac{1}{16}(cos6\theta + 15cos2\theta).$$ |